4 Ratio and Proportion

Problem set 4

(1) Select the appropriate alternative answer for the following questions.

(i) If 6 : 5 = y : 20 then what will be the value of y ?

(A) 15 (B) 24 (C) 18 (D) 22.5

Soln:- $6 : 5 = y : 20$

$\therefore \frac{6}{5} = \frac{y}{20}$

$\therefore y = \frac{6}{\cancel{5}_{1}} \times \cancel{20}^{4}$

$\therefore y = 6 \times 4$

$\therefore \boxed{y = 24}$

Option B

(ii) What is the ratio of 1 mm to 1 cm ?

(A) 1 : 100 (B) 10 : 1 (C) 1 : 10 (D) 100 : 1

Soln:- $1\text{ cm} = 10\text{ mm}$

$\therefore \frac{1\text{ mm}}{1\text{ cm}} = \frac{1\text{ mm}}{10\text{ mm}} = \frac{1}{10} = 1 : 10$

option (c)

(iii*) The ages of Jatin, Nitin and Mohasin are 16, 24 and 36 years respectively. What is the ratio of Nitin's age to Mohasin's age ?

(A) 3 : 2 (B) 2 : 3 (C) 4 : 3 (D) 3 : 4

Soln:-

$$\frac{\text{Nitin's age}}{\text{Mohasin's age}} = \frac{24}{36}$$

$$= \frac{\cancel{12} \times 2}{\cancel{12} \times 3}$$

$$= \frac{2}{3}$$

$$= 2 : 3$$

option (B)

(iv) 24 Bananas were distributed between Shubham and Anil in the ratio 3 : 5, then how many bananas did Shubham get ?

(A) 8 (B) 15 (C) 12 (D) 9

Soln:-

Number of bananas shubham got,

$$= \frac{3}{(3+5)} \times 24$$

$$= \frac{3}{\cancel{8}_{1}} \times \cancel{24}^{3}$$

$= 3 \times 3$

$= 9$

Option D

(v) What is the mean proportional of 4 and 25 ?

(A) 6 (B) 8 (C) 10 (D) 12

Soln :-

Mean proportional,

$b^2 = ac$

$\therefore b^2 = 4 \times 25$

$\therefore b^2 = 100$

$\therefore \boxed{b = 10}$

Option C

(2) For the following numbers write the ratio of first number to second number in the reduced form.

(i) 21, 48 (ii) 36, 90 (iii) 65, 117 (iv) 138, 161 (v) 114, 133

Soln :-

i) 21, 48

$\frac{21}{48} = \frac{\cancel{3} \times 7}{\cancel{3} \times 16} = \frac{7}{16} = 7:16$

ii) 36, 90

$$\frac{36}{90} = \frac{\cancel{18} \times 2}{\cancel{18} \times 5} = \frac{2}{5} = 2:5$$

iii) 65, 117

$$\frac{65}{117} = \frac{\cancel{13} \times 5}{\cancel{13} \times 9} = \frac{5}{9} = 5:9$$

iv) 138, 161

$$\frac{138}{161} = \frac{\cancel{23} \times 6}{\cancel{23} \times 7} = \frac{6}{7} = 6:7$$

v) 114, 133

$$\frac{114}{133} = \frac{\cancel{19} \times 6}{\cancel{19} \times 7} = \frac{6}{7} = 6:7$$

(3) Write the following ratios in the reduced form.

(i) Radius to the diameter of a circle.

(ii) The ratio of diagonal to the length of a rectangle, having length 4 cm and breadth 3 cm.

(iii) The ratio of perimeter to area of a square, having side 4 cm.

Soln:-

i) Radius to the diameter of a circle.

$$\frac{\text{Radius}}{\text{Diameter}} = \frac{r}{d} = \frac{\cancel{r}}{2\cancel{r}} = \frac{1}{2} = 1:2$$

ii) The ratio of diagonal to the length

of a rectangle, having length 4 cm and breadth 3 cm.

Diagonal of a rectangle

$$= \sqrt{l^2 + b^2}$$

$$= \sqrt{(4)^2 + (3)^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5 \text{ cm}$$

$$\frac{\text{Diagonal}}{\text{Length}} = \frac{5}{4} = 5:4$$

iii) The ratio of perimeter to area of a square, having side 4 cm.

$$\frac{\text{Perimeter}}{\text{Area}} = \frac{4 \times \text{side}}{(\text{side})^2}$$

$$= \frac{4 \times 4}{(4)^2}$$

$$= \frac{16}{16}$$

$$= \frac{1}{1}$$

$= 1 : 1$

(4) Check whether the following numbers are in continued proportion.

(i) 2, 4, 8 (ii) 1, 2, 3 (iii) 9, 12, 16 (iv) 3, 5, 8

Soln:-

i) 2, 4, 8

$$\frac{2}{4} = \frac{1}{2}$$

& $$\frac{4}{8} = \frac{1}{2}$$

Here, $\frac{2}{4} = \frac{4}{8}$

$\therefore$ The numbers 2, 4 and 8 are in continued proportion.

ii) 1, 2, 3

$$\frac{1}{2} = \frac{1}{2}$$

& $$\frac{2}{3} = \frac{2}{3}$$

Here, $\frac{1}{2} \neq \frac{2}{3}$

$\therefore$ The numbers 1, 2 and 3 are not in continued proportion.

iii) 9, 12, 16

$$\frac{9}{12} = \frac{\cancel{3} \times 3}{\cancel{3} \times 4} = \frac{3}{4}$$

& $$\frac{12}{16} = \frac{\cancel{4} \times 3}{\cancel{4} \times 4} = \frac{3}{4}$$

Here, $\frac{9}{12} = \frac{12}{16}$

$\therefore$ The numbers 9, 12 and 16 are in continued proportion.

iv) 3, 5, 8

$$\frac{3}{5} = \frac{3}{5}$$

& $$\frac{5}{8} = \frac{5}{8}$$

Here, $\frac{3}{5} \neq \frac{5}{8}$

$\therefore$ The numbers 3, 5 and 8 are not in continued proportion.

(5) a, b, c are in continued proportion. If $a = 3$ and $c = 27$ then find b.

Soln:- Since, a, b, c are in continued proportion.

$$\therefore \quad b^2 = ac$$

$$= 3 \times 27$$

$$b^2 = 81$$

$$\therefore \quad \boxed{b = 9}$$

(6) Convert the following ratios into percentages..

(i) 37 : 500 (ii) $\frac{5}{8}$ (iii) $\frac{22}{30}$ (iv) $\frac{5}{16}$ (v) $\frac{144}{1200}$

Soln:-

i) 37 : 500

$$\frac{37}{\cancel{500}_{5}} \times \cancel{100}^{1} = \frac{37}{5} = 7.4\ \%$$

ii) $\frac{5}{8}$

$$\frac{5}{\cancel{8}_{2}} \times \cancel{100}^{25} = \frac{25 \times 5}{2}$$

$$= \frac{125}{2}$$

$$= 62.5\ \%$$

iii) $\frac{22}{30}$

$\frac{22}{3\cancel{0}} \times 10\cancel{0} = \frac{22 \times 10}{3}$

$= \frac{220}{3}$

$= 73.33\%$

iv) $\frac{5}{16}$

$\frac{5}{\cancel{16}_{4}} \times \cancel{100}^{25} = \frac{25 \times 5}{4}$

$= \frac{125}{4}$

$= 31.25\%$

v) $\frac{144}{1200}$

$\frac{144}{12\cancel{00}} \times 1\cancel{00} = \frac{144}{12}$

$= 12\%$

(7) Write the ratio of first quantity to second quantity in the reduced form.

(i) 1024 MB, 1.2 GB [(1024 MB = 1 GB)]

(ii) 17 Rupees, 25 Rupees 60 paise

(iii) 5 dozen, 120 units

(iv) 4 sq.m, 800 sq.cm

(v) 1.5 kg, 2500 gm

Soln:-

i) 1024 MB , 1.2 GB [1024 MB = 1 GB]

$$\frac{1024 \text{ MB}}{1.2 \text{ GB}} = \frac{\cancel{1024}}{1.2 \times \cancel{1024}}$$

$$= \frac{1}{1.2}$$

$$= \frac{10}{12}$$

$$= \frac{5}{6}$$

$$= 5 : 6$$

ii) 17 rupees , 25 rupees 60 paise

$$\frac{17 \text{ rupees}}{25 \text{ rupees } 60 \text{ paise}} = \frac{17 \times 100}{(25 \times 100) + 60}$$

$$= \frac{1700}{2500 + 60}$$

$$= \frac{170\cancel{0}}{256\cancel{0}}$$

$$= \frac{85}{128}$$

$$= 85 : 128$$

iii) 5 dozen, 120 units

$$\frac{5 \text{ dozen}}{120 \text{ units}} = \frac{5 \times 12}{120}$$

$$= \frac{6\cancel{0}}{12\cancel{0}}$$

$$= \frac{1}{2}$$

$$= 1 : 2$$

iv) 4 sq. m , 800 sq. cm

$$\frac{4 \text{ sq. m}}{800 \text{ sq. cm}} = \frac{\overset{1}{\cancel{4}} \times \overset{50}{\cancel{100}} \times 1\cancel{0}\cancel{0}}{\underset{2}{8}\underset{1}{\cancel{0}}\cancel{0}}$$

$$= \frac{50}{1}$$

$$= 50 : 1$$

v) 1.5 kg , 2500 gm

$$\frac{1.5 \text{ kg}}{2500 \text{ gm}} = \frac{1.5 \times 10\cancel{0}\cancel{0}}{25\cancel{0}\cancel{0}}$$

$$= \frac{1.5 \times 10}{25}$$

$$= \frac{15}{25}$$

$$= \frac{3}{5}$$

$$= 3 : 5$$

(8) If $\frac{a}{b} = \frac{2}{3}$ then find the values of the following expressions.

(i) $\frac{4a+3b}{3b}$ (ii) $\frac{5a^2+2b^2}{5a^2-2b^2}$

(iii) $\frac{a^3+b^3}{b^3}$ (iv) $\frac{7b-4a}{7b+4a}$

Soln:-

i) $$\frac{a}{b} = \frac{2}{3}$$

$$\frac{4}{3} \times \frac{a}{b} = \frac{2}{3} \times \frac{4}{3}$$

$$\frac{4a}{3b} = \frac{8}{9}$$

Applying componendo,

$$\frac{4a+3b}{3b} = \frac{8+9}{9}$$

$$\boxed{\frac{4a+3b}{3b} = \frac{17}{9}}$$

ii) $$\frac{a}{b} = \frac{2}{3}$$

$$\frac{a^2}{b^2} = \left(\frac{2}{3}\right)^2$$

$$\frac{a^2}{b^2} = \frac{4}{9}$$

$$\frac{5}{2} \times \frac{a^2}{b^2} = \frac{5}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{9}$$

$$\frac{5a^2}{2b^2} = \frac{10}{9}$$

Applying componendo & dividendo,

$$\frac{5a^2+2b^2}{5a^2-2b^2} = \frac{10+9}{10-9}$$

$$\frac{5a^2+2b^2}{5a^2-2b^2} = \frac{19}{1}$$

$$\boxed{\frac{5a^2+2b^2}{5a^2-2b^2} = 19}$$

iii) $$\frac{a}{b} = \frac{2}{3}$$

$$\frac{a^3}{b^3} = \left(\frac{2}{3}\right)^3$$

$$\frac{a^3}{b^3} = \frac{8}{27}$$

Applying Componendo,

$$\frac{a^3+b^3}{b^3} = \frac{8+27}{27}$$

$$\boxed{\frac{a^3+b^3}{b^3} = \frac{35}{27}}$$

iv) $$\frac{a}{b} = \frac{2}{3}$$

$$\frac{b}{a} = \frac{3}{2}$$

$$\frac{7}{4} \times \frac{b}{a} = \frac{7}{4} \times \frac{3}{2}$$

$$\frac{7b}{4a} = \frac{21}{8}$$

Applying componendo & dividendo,

$$\frac{7b+4a}{7b-4a} = \frac{21+8}{21-8}$$

$$\frac{7b+4a}{7b-4a} = \frac{29}{13}$$

$$\boxed{\frac{7b-4a}{7b+4a} = \frac{13}{29}}$$

(9) If a, b, c, d are in proportion, then prove that

(i) $$\frac{11a^2+9ac}{11b^2+9bd} = \frac{a^2+3ac}{b^2+3bd}$$

Soln:-

As a, b, c, d are in proportion.

$$\therefore \frac{a}{b} = \frac{c}{d} = K$$

$$\therefore a = bK \quad \& \quad c = dK$$

i) $$\frac{11a^2 + 9ac}{11b^2 + 9bd} = \frac{a^2 + 3ac}{b^2 + 3bd}$$

$$LHS = \frac{11a^2 + 9ac}{11b^2 + 9bd}$$

$$= \frac{11(bK)^2 + 9 \times bK \times dK}{11b^2 + 9 \times b \times d}$$

$$= \frac{11b^2K^2 + 9bdK^2}{11b^2 + 9bd}$$

$$= \frac{K^2 \cancel{(11b^2 + 9bd)}}{\cancel{11b^2 + 9bd}}$$

$$LHS = K^2 \quad \text{———} \quad (I)$$

And,

$$RHS = \frac{a^2 + 3ac}{b^2 + 3bd}$$

$$= \frac{(bK)^2 + 3 \times bK \times dK}{b^2 + 3bd}$$

$$= \frac{b^2k^2 + 3bdk^2}{b^2 + 3bd}$$

$$= \frac{k^2 \cancel{(b^2 + 3bd)}}{\cancel{b^2 + 3bd}}$$

$$RHS = k^2 \text{ —— } \text{II}$$

from I and II,

$$LHS = RHS$$

$$\frac{11a^2 + 9ac}{11b^2 + 9bd} = \frac{a^2 + 3ac}{b^2 + 3bd}$$

(ii*) $\sqrt{\dfrac{a^2 + 5c^2}{b^2 + 5d^2}} = \dfrac{a}{b}$

<u>Soln</u>:- $LHS = \sqrt{\dfrac{a^2 + 5c^2}{b^2 + 5d^2}}$

$$= \sqrt{\frac{(bk)^2 + 5(dk)^2}{b^2 + 5d^2}}$$

$$= \sqrt{\frac{b^2k^2 + 5d^2k^2}{b^2 + 5d^2}}$$

$$= \sqrt{\frac{k^2 \cancel{(b^2 + 5d^2)}}{\cancel{b^2 + 5d^2}}}$$

$$= \sqrt{k^2}$$

$$LHS = k \text{ —— } \text{I}$$

$$RHS = \frac{a}{b}$$

$$= \frac{\cancel{b}k}{\cancel{b}}$$

$$\therefore \quad RHS = k \quad \text{———} \quad (II)$$

from (I) & (II)

$$LHS = RHS$$

$$\sqrt{\frac{a^2 + 5c^2}{b^2 + 5d^2}} = \frac{a}{b}$$

(iii) $\dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{c^2+cd+d^2}{c^2-cd+d^2}$

Soln:-

$$LHS = \frac{a^2 + ab + b^2}{a^2 - ab + b^2}$$

$$= \frac{(bk)^2 + bk \times b + b^2}{(bk)^2 - bk \times b + b^2}$$

$$= \frac{b^2k^2 + b^2k + b^2}{b^2k^2 - b^2k + b^2}$$

$$= \frac{\cancel{b^2}(k^2+k+1)}{\cancel{b^2}(k^2-k+1)}$$

$$LHS = \frac{k^2+k+1}{k^2-k+1} \quad \text{———} \quad (I)$$

And,

$$RHS = \frac{c^2 + cd + d^2}{c^2 - cd + d^2}$$

$$= \frac{(dK)^2 + dK \times d + d^2}{(dK)^2 - dK \times d + d^2}$$

$$= \frac{d^2K^2 + d^2K + d^2}{d^2K^2 - d^2K + d^2}$$

$$= \frac{\cancel{d^2}(K^2 + K + 1)}{\cancel{d^2}(K^2 - K + 1)}$$

$$RHS = \frac{K^2 + K + 1}{K^2 - K + 1}$$

from I and II,

LHS = RHS

$$\frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{c^2 + cd + d^2}{c^2 - cd + d^2}$$

(10) If a, b, c are in continued proportion, then prove that

(i) $\dfrac{a}{a+2b} = \dfrac{a-2b}{a-4c}$ (ii) $\dfrac{b}{b+c} = \dfrac{a-b}{a-c}$

Soln:- As a, b, c are in continued proportion.

$$\therefore \frac{a}{b} = \frac{b}{c} = K$$

$\therefore \quad a = bk \quad \& \quad b = ck$

$a = ck \times k = ck^2$

i) $\dfrac{a}{a+2b} = \dfrac{a-2b}{a-4c}$

$$\text{LHS} = \frac{a}{a+2b}$$

$$= \frac{ck^2}{ck^2 + 2ck}$$

$$= \frac{\cancel{c}k^{\cancel{2}}}{\cancel{c}\cancel{k}(k+2)}$$

$$\text{LHS} = \frac{k}{k+2} \quad \text{———} \quad \text{I}$$

And,

$$\text{RHS} = \frac{a-2b}{a-4c}$$

$$= \frac{ck^2 - 2ck}{ck^2 - 4c}$$

$$= \frac{\cancel{c}k(k-2)}{\cancel{c}(k^2-4)}$$

$$= \frac{k\cancel{(k-2)}}{(k+2)\cancel{(k-2)}}$$

$$RHS = \frac{K}{K+2} \text{ ———— } (\mathrm{II})$$

from (I) and (II),

$$LHS = RHS$$

$$\frac{a}{a+2b} = \frac{a-2b}{a-4c}$$

ii) $$\frac{b}{b+c} = \frac{a-b}{a-c}$$

$$LHS = \frac{b}{b+c}$$

$$= \frac{ck}{ck+c}$$

$$= \frac{\cancel{c}k}{\cancel{c}(k+1)}$$

$$LHS = \frac{K}{K+1} \text{ ———— } (\mathrm{I})$$

And,

$$RHS = \frac{a-b}{a-c}$$

$$= \frac{ck^2-ck}{ck^2-c}$$

$$= \frac{\cancel{2}K(K-1)}{\cancel{2}(K^2-1)}$$

$$= \frac{K(K\cancel{-1})}{(K\cancel{-1})(K+1)}$$

$$RHS = \frac{K}{K+1} \quad \text{—— II}$$

from I and II,

LHS = RHS

$$\frac{b}{b+c} = \frac{a-b}{a-c}$$

(11) Solve : $\dfrac{12x^2+18x+42}{18x^2+12x+58} = \dfrac{2x+3}{3x+2}$

Soln:-

$$\frac{12x^2+18x+42}{18x^2+12x+58} = \frac{2x+3}{3x+2}$$

If $x = 0$ then,

$$\frac{12\times(0)^2+18\times0+42}{18\times(0)^2+12\times0+58} = \frac{2\times0+3}{3\times0+2}$$

$$\frac{42}{58} \neq \frac{3}{2}$$

$\therefore$ $x = 0$ is not the solution of this eq^n

Now,

$$\frac{12x^2+18x+42}{18x^2+12x+58} = \frac{2x+3}{3x+2}$$

$$\frac{12x^2+18x+42}{18x^2+12x+58} = \frac{6x(2x+3)}{6x(3x+2)}$$

$$\frac{12x^2+18x+42}{18x^2+12x+58} = \frac{12x^2+18x}{18x^2+12x}$$

By using the theorem of equal ratios,

$$= \frac{12x^2+18x+42-(12x^2+18x)}{18x^2+12x+58-(18x^2+12x)}$$

$$= \frac{\cancel{12x^2}+\cancel{18x}+42-\cancel{12x^2}-\cancel{18x}}{\cancel{18x^2}+\cancel{12x}+58-\cancel{18x^2}-\cancel{12x}}$$

$$= \frac{42}{58}$$

$$\therefore \quad \frac{2x+3}{3x+2} = \frac{42}{58}$$

$$\therefore \quad \frac{2x+3}{3x+2} = \frac{21}{29}$$

$$\therefore \quad 29(2x+3) = 21(3x+2)$$

$$\therefore \quad 58x+87 = 63x+42$$

$$\therefore \quad 63x-58x = 87-42$$

$\therefore \quad 5x = 45$

$\therefore \quad x = \dfrac{45}{5}$

$\therefore \quad \boxed{x = 9}$

$\therefore$ $x = 9$ is the solution of the given eqⁿ

(12) If $\dfrac{2x-3y}{3z+y} = \dfrac{z-y}{z-x} = \dfrac{x+3z}{2y-3x}$ then prove that every ratio $= \dfrac{x}{y}$.

Solⁿ :-

$$\frac{2x-3y}{3z+y} = \frac{z-y}{z-x} = \frac{x+3z}{2y-3x}$$

$$\frac{2x-3y}{3z+y} = \frac{3(z-y)}{3(z-x)} = \frac{x+3z}{2y-3x}$$

$$\frac{2x-3y}{3z+y} = \frac{3z-3y}{3z-3x} = \frac{x+3z}{2y-3x}$$

By the theorem of equal ratios,

$$= \frac{2x-3y-(3z-3y)+x+3z}{3z+y-(3z-3x)+2y-3x}$$

$$= \frac{2x-\cancel{3y}-\cancel{3z}+\cancel{3y}+x+\cancel{3z}}{\cancel{3z}+y-\cancel{3z}+\cancel{3x}+2y-\cancel{3x}}$$

$$= \frac{\cancel{3}x}{\cancel{3}y}$$

$$= \frac{x}{y}$$

(13*) If $\dfrac{by+cz}{b^2+c^2} = \dfrac{cz+ax}{c^2+a^2} = \dfrac{ax+by}{a^2+b^2}$ then prove that $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$.

Soln:-

$$\frac{by+cz}{b^2+c^2} = \frac{cz+ax}{c^2+a^2} = \frac{ax+by}{a^2+b^2}$$

By the theorem of equal ratios,

$$= \frac{by+cz+cz+ax+ax+by}{b^2+c^2+c^2+a^2+a^2+b^2}$$

$$= \frac{2ax+2by+2cz}{2a^2+2b^2+2c^2}$$

$$= \frac{\cancel{2}(ax+by+cz)}{\cancel{2}(a^2+b^2+c^2)}$$

$$= \frac{ax+by+cz}{a^2+b^2+c^2}$$

By the theorem of equal ratios,

$$\frac{by+cz-(ax+by+cz)}{b^2+c^2-(a^2+b^2+c^2)} = \frac{cz+ax-(ax+by+cz)}{c^2+a^2-(a^2+b^2+c^2)}$$

$$= \frac{ax+by-(ax+by+cz)}{a^2+b^2-(a^2+b^2+c^2)}$$

$$\frac{\cancel{by} + \cancel{cz} - ax - \cancel{by} - \cancel{cz}}{\cancel{b^2} + \cancel{c^2} - a^2 - \cancel{b^2} - \cancel{c^2}} = \frac{\cancel{cz} + \cancel{ax} - \cancel{ax} - by - \cancel{cz}}{\cancel{c^2} + \cancel{a^2} - \cancel{a^2} - b^2 - \cancel{c^2}}$$

$$= \frac{\cancel{ax} + \cancel{by} - \cancel{ax} - \cancel{by} - cz}{\cancel{a^2} + \cancel{b^2} - \cancel{a^2} - \cancel{b^2} - c^2}$$

$$\therefore \quad \frac{+\cancel{a}x}{+a^{\cancel{2}}} = \frac{+\cancel{b}y}{+b^{\cancel{2}}} = \frac{+\cancel{c}z}{+c^{\cancel{2}}}$$

$$\therefore \quad \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$