3 Polynomials

Practice set 3.2

(1) Use the given letters to write the answer.

(i) There are 'a' trees in the village Lat. If the number of trees increases every year by 'b', then how many trees will there be after 'x' years?

(ii) For the parade there are y students in each row and x such row are formed. Then, how many students are there for the parade in all ?

(iii) The tens and units place of a two digit number is m and n respectively. Write the polynomial which represents the two digit number.

Soln:-

i) Number of trees after 'x' years

$$= \left(\begin{array}{c}\text{Initial number}\\ \text{of trees}\end{array}\right) + \left(\begin{array}{c}\text{Number of}\\ \text{years}\end{array}\right) \times \left(\begin{array}{c}\text{Increase}\\ \text{every year}\end{array}\right)$$

$$= a + bx$$

ii) Number of students in a parade

$$= \left(\begin{array}{c}\text{Number of}\\ \text{rows}\end{array}\right) \times \left(\begin{array}{c}\text{Number of students}\\ \text{in each row}\end{array}\right)$$

$$= xy$$

iii) Two digit number

= (Tens place Number) + (Unit place Number)

$= 10 \times m + n$

$= 10m + n$

(2) Add the given polynomials.

(i) $x^3 - 2x^2 - 9$; $5x^3 + 2x + 9$

(ii) $-7m^4 + 5m^3 + \sqrt{2}$; $5m^4 - 3m^3 + 2m^2 + 3m - 6$

(iii) $2y^2 + 7y + 5$; $3y + 9$; $3y^2 - 4y - 3$

Soln:-

i) $x^3 - 2x^2 - 9$; $5x^3 + 2x + 9$

$= x^3 - 2x^2 - \cancel{9} + 5x^3 + 2x + \cancel{9}$

$= 6x^3 - 2x^2 + 2x$

ii) $-7m^4 + 5m^3 + \sqrt{2}$; $5m^4 - 3m^3 + 2m^2 + 3m - 6$

$= -7m^4 + 5m^3 + \sqrt{2} + 5m^4 - 3m^3 + 2m^2 + 3m - 6$

$= -2m^4 + 2m^3 + 2m^2 + 3m + \sqrt{2} - 6$

iii) $2y^2 + 7y + 5$; $3y + 9$; $3y^2 - 4y - 3$

$= 2y^2 + 7y + 5 + 3y + 9 + 3y^2 - 4y - 3$

$= 5y^2 + 6y + 11$

(3) Subtract the second polynomial from the first.

(i) $x^2 - 9x + \sqrt{3}$; $-19x + \sqrt{3} + 7x^2$

(ii) $2ab^2 + 3a^2b - 4ab$; $3ab - 8ab^2 + 2a^2b$

Soln:-

i) $x^2 - 9x + \sqrt{3}$; $-19x + \sqrt{3} + 7x^2$

$= (x^2 - 9x + \sqrt{3}) - (-19x + \sqrt{3} + 7x^2)$

$= x^2 - 9x + \cancel{\sqrt{3}} + 19x - \cancel{\sqrt{3}} - 7x^2$

$= -6x^2 + 10x$

ii) $2ab^2 + 3a^2b - 4ab$; $3ab - 8ab^2 + 2a^2b$

$= (2ab^2 + 3a^2b - 4ab) - (3ab - 8ab^2 + 2a^2b)$

$= 2ab^2 + 3a^2b - 4ab - 3ab + 8ab^2 - 2a^2b$

$= 10ab^2 + a^2b - 7ab$

(4) Multiply the given polynomials.

(i) $2x$; $x^2 - 2x - 1$ (ii) $x^5 - 1$; $x^3 + 2x^2 + 2$ (iii) $2y + 1$; $y^2 - 2y^3 + 3y$

Soln:-

i) $2x$; $x^2 - 2x - 1$

$= 2x(x^2 - 2x - 1)$

$= 2x \times x^2 - 2x \times 2x - 2x \times 1$

$= 2x^3 - 4x^2 - 2x$

ii) $x^5 - 1 \; ; \; x^3 + 2x^2 + 2$

$= (x^5 - 1)(x^3 + 2x^2 + 2)$

$= x^5 (x^3 + 2x^2 + 2) - 1 (x^3 + 2x^2 + 2)$

$= x^5 \times x^3 + x^5 \times 2x^2 + 2 \times x^5 - 1 \times x^3 - 1 \times 2x^2 - 1 \times 2$

$= x^8 + 2x^7 + 2x^5 - x^3 - 2x^2 - 2$

iii) $2y + 1 \; ; \; y^2 - 2y^3 + 3y$

$= (2y + 1)(y^2 - 2y^3 + 3y)$

$= 2y (y^2 - 2y^3 + 3y) + 1 (y^2 - 2y^3 + 3y)$

$= 2y \times y^2 - 2y \times 2y^3 + 2y \times 3y + 1 \times y^2 + 1 \times -2y^3 + 1 \times 3y$

$= \cancel{2y^3} - 4y^4 + 6y^2 + y^2 - \cancel{2y^3} + 3y$

$= -4y^4 + 7y^2 + 3y$

(5) Divide first polynomial by second polynomial and write the answer in the form 'Dividend = Divisor × Quotient + Remainder'.

(i) $x^3 - 64; \; x - 4$ (ii) $5x^5 + 4x^4 - 3x^3 + 2x^2 + 2; \; x^2 - x$

Soln:- i) $x^3 - 64$; $x - 4$

$$
\begin{array}{r|l}
 & x^2 + 4x + 16 \\
\hline
x - 4 & \cancel{x^3} - 64 \\
 & -\ \cancel{x^3} - 4x^2 \\
 & \quad - \qquad + \\
\hline
 & \cancel{4x^2} - 64 \\
 & -\ \cancel{4x^2} - 16x \\
 & \quad - \qquad + \\
\hline
 & 16\cancel{x} - 6\cancel{4} \\
 & -\ 1\cancel{6}x - 6\cancel{4} \\
 & \quad - \qquad + \\
\hline
 & 0
\end{array}
$$

Dividend = (Divisor × Quotient) + Remainder

$$x^3 - 64 = (x - 4)(x^2 + 4x + 16) + 0$$

ii) $5x^5 + 4x^4 - 3x^3 + 2x^2 + 2$; $x^2 - x$

$$
\begin{array}{r|l}
 & 5x^3 + 9x^2 + 6x + 8 \\
\hline
x^2 - x & 5\cancel{x^5} + 4x^4 - 3x^3 + 2x^2 + 2 \\
 & -\ 5\cancel{x^5} - 5x^4 \\
 & \quad - \qquad + \\
\hline
 & 9\cancel{x^4} - 3x^3 + 2x^2 + 2 \\
 & -\ 9\cancel{x^4} - 9x^3 \\
 & \quad - \qquad + \\
\hline
 & 6\cancel{x^3} + 2x^2 + 2 \\
 & -\ 6\cancel{x^3} - 6x^2 \\
 & \quad - \qquad + \\
\hline
 & 8\cancel{x^2} + 2
\end{array}
$$

$$-\cancel{8x^2} - 8x$$
$$\quad - \quad\; +$$

$$8x + 2$$

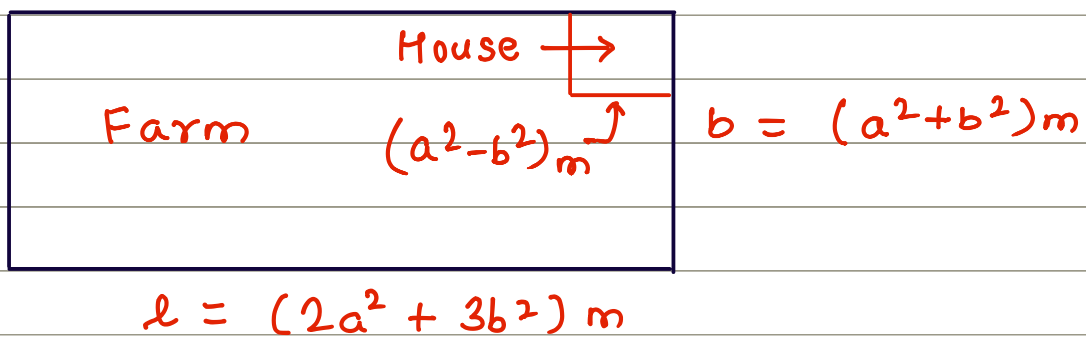
Dividend = (Divisor × Quotient) + Remainder

$$5x^5 + 4x^4 - 3x^3 + 2x^2 + 2$$

$$= (x^2 - x)\,(5x^3 + 9x^2 + 6x + 8) + (8x + 2)$$

(6*) Write down the information in the form of algebraic expression and simplify. There is a rectangular farm with length ($2a^2 + 3b^2$) metre and breadth ($a^2 + b^2$) metre. The farmer used a square shaped plot of the farm to build a house. The side of the plot was ($a^2 - b^2$) metre. What is the area of the remaining part of the farm ?

Solⁿ:-



Area of the rectangular farm

$$= l \times b$$

$$= (2a^2 + 3b^2)\,(a^2 + b^2)$$

$$= 2a^2\,(a^2 + b^2) + 3b^2\,(a^2 + b^2)$$

$$= 2a^2 \times a^2 + 2a^2 \times b^2 + 3b^2 \times a^2 + 3b^2 \times b^2$$

$= 2a^4 + 2a^2b^2 + 3a^2b^2 + 3b^4$

$= (2a^4 + 5a^2b^2 + 3b^4)\ m^2$

Area of the square shaped plot,

$= (side)^2$

$= (a^2 - b^2)^2$

$= (a^2)^2 - (2 \times a^2 \times b^2) + (b^2)^2$

$= (a^4 - 2a^2b^2 + b^4)\ m^2$

$\therefore$ Area of the remaining part of the farm

$$= \left[\begin{array}{c}\text{Area of the} \\ \text{Rectangular} \\ \text{farm}\end{array}\right] - \left[\begin{array}{c}\text{Area of the} \\ \text{square shaped} \\ \text{plot}\end{array}\right]$$

$= (2a^4 + 5a^2b^2 + 3b^4) - (a^4 - 2a^2b^2 + b^4)$

$= 2a^4 + 5a^2b^2 + 3b^4 - a^4 + 2a^2b^2 - b^4$

$= (3a^4 + 7a^2b^2 + 2b^4)\ m^2$