2 Quadratic Equations

Practice Set 2.3

Solve the following quadratic equations by completing the square method.

(1) $x^2 + x - 20 = 0$

Soln:- Here,

$2a = 1$ --- (coefficient of x)

$\therefore \quad a = \frac{1}{2}$

$\therefore \quad a^2 = \frac{1}{4}$

$\therefore \quad x^2 + x - 20 = 0$

$\therefore \quad x^2 + x + \frac{1}{4} - \frac{1}{4} - 20 = 0$

$\therefore \quad \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{4} + 20\right) = 0$

$\therefore \quad \left(x + \frac{1}{2}\right)^2 - \frac{81}{4} = 0$

$\therefore \quad \left(x + \frac{1}{2}\right)^2 = \frac{81}{4}$

$\therefore \quad \left(x + \frac{1}{2}\right) = \pm\sqrt{\frac{81}{4}}$

$$\therefore \quad \left(x + \frac{1}{2}\right) = \pm \frac{9}{2}$$

$$\therefore \quad x = -\frac{1}{2} + \frac{9}{2} \quad \text{or} \quad x = -\frac{1}{2} - \frac{9}{2}$$

$$\therefore \quad x = \frac{-1+9}{2} \quad \text{or} \quad x = \frac{-1-9}{2}$$

$$\therefore \quad x = \frac{8}{2} \quad \text{or} \quad x = \frac{-10}{2}$$

$$\therefore \quad \boxed{x = 4} \quad \text{or} \quad \boxed{x = -5}$$

$\therefore$ $x = 4$ & $x = -5$ are the roots of the given quadratic equation.

(2) $x^2 + 2x - 5 = 0$

Soln:- Here,

$$2a = 2 \quad \text{--- (coeff. of } x\text{)}$$

$$\therefore \quad a = \frac{2}{2}$$

$$\therefore \quad a = 1$$

$$\therefore \quad a^2 = 1$$

$\therefore \quad x^2 + 2x - 5 = 0$

$\therefore \quad x^2 + 2x + 1 - 1 - 5 = 0$

$\therefore \quad (x+1)^2 - 6 = 0$

$\therefore \quad (x+1)^2 = 6$

$\therefore \quad (x+1) = \pm\sqrt{6}$

$\therefore \quad x = -1 + \sqrt{6} \quad$ or $\quad x = -1 - \sqrt{6}$

$\therefore x = -1+\sqrt{6}$ & $x = -1-\sqrt{6}$ are the roots of the given quadratic equation.

(3) $m^2 - 5m = -3$

Sol^n:- $m^2 - 5m = -3$

$\therefore \quad m^2 - 5m + 3 = 0$

Now, $2a = -5$ --- (coeff. of x)

$\therefore \quad a = \frac{-5}{2}$

$\therefore \quad a^2 = \frac{25}{4}$

$\therefore \quad m^2 - 5m + 3 = 0$

$\therefore \quad m^2 - 5m + \frac{25}{4} - \frac{25}{4} + 3 = 0$

$\therefore \quad \left(m - \frac{5}{2}\right)^2 = \frac{25}{4} - 3$

$\therefore \quad \left(m - \frac{5}{2}\right)^2 = \frac{25 - 12}{4}$

$\therefore \quad \left(m - \frac{5}{2}\right)^2 = \frac{13}{4}$

$\therefore \quad m - \frac{5}{2} = \pm \frac{\sqrt{13}}{2}$

$\therefore \quad m = \frac{5}{2} + \frac{\sqrt{13}}{2} \quad \text{or} \quad m = \frac{5}{2} - \frac{\sqrt{13}}{2}$

$\therefore \quad m = \frac{5+\sqrt{13}}{2} \quad \text{or} \quad m = \frac{5-\sqrt{13}}{2}$

$\therefore \quad m = \frac{5+\sqrt{13}}{2}$ & $m = \frac{5-\sqrt{13}}{2}$ are the roots of the given quadratic equation.

(4) $9y^2 - 12y + 2 = 0$

Soln:- $9y^2 - 12y + 2 = 0$

$$\therefore \quad \frac{9y^2}{9} - \frac{12y}{9} + \frac{2}{9} = 0$$

$$\therefore \quad y^2 - \frac{12}{9}y + \frac{2}{9} = 0$$

$$\therefore \quad y^2 - \frac{4}{3}y + \frac{2}{9} = 0$$

Here, $\quad 2a = \frac{-4}{3}$

$$\therefore \quad a = \frac{-4}{3 \times 2}$$

$$\therefore \quad a = \frac{-4}{6}$$

$$\therefore \quad a = \frac{-2}{3}$$

$$\therefore \quad a^2 = \frac{4}{9}$$

$$\therefore \quad y^2 - \frac{4}{3}y + \frac{2}{9} = 0$$

$$\therefore \quad y^2 - \frac{4}{3}y + \frac{4}{9} - \frac{4}{9} + \frac{2}{9} = 0$$

$$\therefore \quad \left(y - \frac{2}{3}\right)^2 = \frac{4}{9} - \frac{2}{9}$$

$$\therefore \quad \left(y - \frac{2}{3}\right)^2 = \frac{4-2}{9}$$

$$\therefore \quad \left(y - \frac{2}{3}\right)^2 = \frac{2}{9}$$

$$\therefore \quad y - \frac{2}{3} = \pm \frac{\sqrt{2}}{3}$$

$$\therefore \quad y = \frac{2}{3} + \frac{\sqrt{2}}{3} \quad \text{or} \quad y = \frac{2}{3} - \frac{\sqrt{2}}{3}$$

$$\therefore \quad y = \frac{2+\sqrt{2}}{3} \quad \text{or} \quad y = \frac{2-\sqrt{2}}{3}$$

$\therefore y = \frac{2+\sqrt{2}}{3}$ & $y = \frac{2-\sqrt{2}}{3}$ are the roots of the given quadratic equation.

(5) $2y^2 + 9y + 10 = 0$

Soln:-

$$2y^2 + 9y + 10 = 0$$

$$\therefore \quad \frac{2y^2}{2} + \frac{9}{2}y + \frac{10}{2} = 0$$

$$\therefore \quad y^2 + \frac{9}{2}y + 5 = 0$$

Here, $2a = \frac{9}{2}$

$$\therefore \quad a = \frac{9}{2 \times 2}$$

$$\therefore \quad a = \frac{9}{4}$$

$$\therefore \quad a^2 = \left(\frac{9}{4}\right)^2$$

$$\therefore \quad a^2 = \frac{81}{16}$$

$$\therefore \quad y^2 + \frac{9}{2}y + 5 = 0$$

$$\therefore \quad y^2 + \frac{9}{2}y + \frac{81}{16} - \frac{81}{16} + 5 = 0$$

$$\therefore \quad \left(y + \frac{9}{4}\right)^2 = \frac{81}{16} - 5$$

$$\therefore \quad \left(y + \frac{9}{4}\right)^2 = \frac{81 - (16 \times 5)}{16}$$

$$\therefore \quad \left(y + \frac{9}{4}\right)^2 = \frac{81 - 80}{16}$$

$$\therefore \quad \left(y + \frac{9}{4}\right)^2 = \frac{1}{16}$$

$$\therefore \quad y + \frac{9}{4} = \pm \frac{1}{4}$$

$\therefore \quad y = -\frac{9}{4} + \frac{1}{4} \quad \text{or} \quad y = \frac{-9}{4} - \frac{1}{4}$

$\therefore \quad y = \frac{-9+1}{4} \quad \text{or} \quad y = \frac{-9-1}{4}$

$\therefore \quad y = \frac{-8}{4} \quad \text{or} \quad y = \frac{-10}{4}$

$\therefore \quad y = -2 \quad \text{or} \quad y = \frac{-5}{2}$

$\therefore$ $y = -2$ & $y = \frac{-5}{2}$ are the roots of the given quadratic equation.

(6) $5x^2 = 4x + 7$

Soln:-

$5x^2 = 4x + 7$

$\therefore \quad 5x^2 - 4x - 7 = 0$

$\therefore \quad \frac{5x^2}{5} - \frac{4}{5}x - \frac{7}{5} = 0$

$\therefore \quad x^2 - \frac{4}{5}x - \frac{7}{5} = 0$

Here, $2a = \frac{-4}{5}$

$$\therefore \quad a = \frac{-4}{2 \times 5}$$

$$\therefore \quad a = \frac{-4}{10}$$

$$\therefore \quad a = \frac{-2}{5}$$

$$\therefore \quad a^2 = \frac{4}{25}$$

$$\therefore \quad x^2 - \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} - \frac{7}{5} = 0$$

$$\therefore \quad \left(x - \frac{2}{5}\right)^2 = \frac{7}{5} + \frac{4}{25}$$

$$\textcircled{5} \qquad \textcircled{1}$$

$$\therefore \quad \left(x - \frac{2}{5}\right)^2 = \frac{35 + 4}{25}$$

$$\therefore \quad \left(x - \frac{2}{5}\right)^2 = \frac{39}{25}$$

$$\therefore \quad \left(x - \frac{2}{5}\right) = \pm \frac{\sqrt{39}}{5}$$

$$\therefore \quad x = \frac{2}{5} + \frac{\sqrt{39}}{5} \quad \text{or} \quad x = \frac{2}{5} - \frac{\sqrt{39}}{5}$$

$\therefore \quad x = \frac{2+\sqrt{39}}{5} \quad \underline{\text{or}} \quad x = \frac{2-\sqrt{39}}{5}$

$\therefore x = \frac{2+\sqrt{39}}{5}$ & $x = \frac{2-\sqrt{39}}{5}$ are the roots of the given quadratic equation.