1 ▸ Linear Equations in Two Variables

1. Solve the following simultaneous equations.

(1) $\dfrac{2}{x} - \dfrac{3}{y} = 15$; $\dfrac{8}{x} + \dfrac{5}{y} = 77$

Sol^n:-

$$\dfrac{2}{x} - \dfrac{3}{y} = 15 \qquad - \text{(I)}$$

$$\& \quad \dfrac{8}{x} + \dfrac{5}{y} = 77 \qquad - \text{(II)}$$

Let, $\quad \dfrac{1}{x} = m$ $\quad \& \quad \dfrac{1}{y} = n$

∴ Eq^n (I) becomes,

$$2m - 3n = 15 \qquad - \text{(III)}$$

& Eq^n (II) becomes,

$$8m + 5n = 77 \qquad - \text{(IV)}$$

Multiply eq^n (III) by 5,

∴ $\quad 5 \times 2m - 5 \times 3n = 5 \times 15$

∴ $\quad 10m - 15n = 75 \qquad - \text{(V)}$

∴ multiply eqn ④ by 3,

∴ $3 \times 8m + 3 \times 5n = 3 \times 77$

∴ $24m + 15n = 231$ — ⑥

Add eqn ⑤ & eqn ⑥,

$$10m - 15n = 75$$
$$+ \quad 24m + 15n = 231$$
_____
$$34m \qquad\qquad = 306$$

∴ $m = \dfrac{306}{34}$

∴ $\boxed{m = 9}$

Put $m = 9$ in eqn ③,

∴ $2m - 3n = 15$

∴ $(2 \times 9) - 3n = 15$

        $18 - 15 = 3n$

∴ $3 = 3n$

$$\therefore \qquad n = \frac{3}{3}$$

$$\therefore \qquad \boxed{n = 1}$$

Now,

$$m = \frac{1}{x} \qquad \& \qquad n = \frac{1}{y}$$

$$\therefore \qquad 9 = \frac{1}{x} \qquad \& \qquad 1 = \frac{1}{y}$$

$$\therefore \qquad \boxed{x = \frac{1}{9}} \qquad \& \qquad \boxed{y = 1}$$

$\therefore$ The solution is,

$$(x, y) = \left(\frac{1}{9}, 1 \right).$$

(2) $\dfrac{10}{x+y} + \dfrac{2}{x-y} = 4$; $\dfrac{15}{x+y} - \dfrac{5}{x-y} = -2$

Sol^n :-

$$\frac{10}{x+y} + \frac{2}{x-y} = 4 \qquad —①$$

$$\& \quad \frac{15}{x+y} - \frac{5}{x-y} = -2 \qquad —②$$

Let,

$$\frac{1}{x+y} = m \quad \& \quad \frac{1}{x-y} = n$$

∴   Eq. ① becomes,

$$10m + 2n = 4 \quad - ③$$

& Eq. ② becomes,

$$15m - 5n = -2 \quad - ④$$

Multiply eq. ③ by 5,

∴ $5 \times 10m + 5 \times 2n = 5 \times 4$

∴ $50m + 10n = 20 \quad - ⑤$

Multiply eq. ④ by 2,

$$2 \times 15m - 2 \times 5n = -2 \times 2$$

∴ $30m - 10n = -4 \quad - ⑥$

Now, Add eq. ⑤ & eq. ⑥,

$$50m + 10\cancel{n} = 20$$
$$+ \quad 30m - 10\cancel{n} = -4$$

$$80m = 16$$

$$\therefore \quad m = \frac{16}{80}$$

$$\boxed{m = \frac{1}{5}}$$

Put $m = \frac{1}{5}$ in eq? $\underline{\text{(III)}}$,

$$\therefore \quad 10m + 2n = 4$$

$$\therefore \quad \left(10 \times \frac{1}{5}\right) + 2n = 4$$

$$\therefore \quad 2 + 2n = 4$$

$$\therefore \quad 2n = 4 - 2$$

$$\therefore \quad 2n = 2$$

$$\therefore \quad n = \frac{2}{2}$$

$$\boxed{n = 1}$$

Now,

$$m = \frac{1}{x+y} \quad \& \quad n = \frac{1}{x-y}$$

$$\frac{1}{5} = \frac{1}{x+y} \quad \& \quad 1 = \frac{1}{x-y}$$

$$\therefore \quad x + y = 5 \quad - \text{(VII)}$$

$$\& \quad x - y = 1 \quad - \text{(VIII)}$$

Add eqⁿ (VII) & eqⁿ (VIII),

$$\therefore \quad + \begin{array}{r} x + y = 5 \\ x - y = 1 \\ \hline 2x = 6 \end{array}$$

$$\therefore \quad x = \frac{6}{2}$$

$$\therefore \quad \boxed{x = 3}$$

Put $x = 3$ in eqⁿ (VII),

$$\therefore \quad x + y = 5$$

$$\therefore \qquad 3 + y = 5$$

$$\therefore \qquad y = 5 - 3$$

$$\boxed{y = 2}$$

$$\therefore \quad \text{The solution is,}$$

$$(x, y) = (3, 2).$$

(3) $\dfrac{27}{x-2} + \dfrac{31}{y+3} = 85$; $\dfrac{31}{x-2} + \dfrac{27}{y+3} = 89$

Solⁿ:-

$$\frac{27}{x-2} + \frac{31}{y+3} = 85 \qquad -①$$

$$\frac{31}{x-2} + \frac{27}{y+3} = 89 \qquad -②$$

Put, $\dfrac{1}{x-2} = m$ & $\dfrac{1}{y+3} = n$,

$\therefore$ Eqⁿ ① becomes,

$$27m + 31n = 85 \qquad -③$$

& Eqⁿ ② becomes,

$$31m + 27n = 89 \quad - \text{(IV)}$$

Add eqⁿ (III) & eqⁿ (IV),

$$27m + 31n = 85$$
$$+$$
$$31m + 27n = 89$$

_____

$$58m + 58n = 174$$

$$\therefore \quad \frac{58m}{58} + \frac{58n}{58} = \frac{174}{58}$$

$$\therefore \quad m + n = 3 \quad - \text{(V)}$$

Now, substract eqⁿ (III) from eqⁿ (IV),

$$31m + 27n = 89$$
$$-$$
$$27m + 31n = 85$$

$$\underline{ \, - \quad\quad - \quad\quad -}$$

_____

$$4m - 4n = 4$$

$$\therefore \quad \frac{4m}{4} - \frac{4n}{4} = \frac{4}{4}$$

$$\therefore \quad m - n = 1 \quad - \text{(VI)}$$

Add eq? (v) & eq? (vi),

$$m + n = 3$$
$$+ \quad m - n = 1$$
$$\overline{\quad 2m \qquad = 4 \quad}$$

$$\therefore \quad m = \frac{4}{2}$$

$$\therefore \quad \boxed{m = 2}$$

Put $m = 2$ in eq? (v),

$$m + n = 3$$

$$\therefore \quad 2 + n = 3$$

$$\therefore \quad n = 3 - 2$$

$$\therefore \quad \boxed{n = 1}$$

But, $\quad m = \dfrac{1}{x - 2} \quad$ & $\quad n = \dfrac{1}{y + 3}$

$$\therefore \quad 2 = \frac{1}{x - 2} \quad \text{&} \quad 1 = \frac{1}{y + 3}$$

$$\therefore \quad 2(x-2) = 1 \qquad \& \quad y+3 = 1$$

$$\therefore \quad 2x - 4 = 1 \qquad \& \qquad y = 1-3$$

$$\therefore \quad 2x = 1 + 4 \qquad \& \qquad \boxed{y = -2}$$

$$\therefore \quad \boxed{x = \dfrac{5}{2}}$$

$\therefore$ The solution is,

$$(x, y) = \left(\dfrac{5}{2}, -2\right)$$

(4) $\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4}$; $\dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = -\dfrac{1}{8}$

<u>Sol^n:-</u>

$$\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4} \quad -\text{\textcircled{I}}$$

$$\& \quad \dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = -\dfrac{1}{8} \quad -\text{\textcircled{II}}$$

Put, $\dfrac{1}{3x+y} = m$ $\&$ $\dfrac{1}{3x-y} = n$

$\therefore$ Eq^n ① becomes,

$$m + n = \frac{3}{4}$$

Multiply each term by 4,

$\therefore$ $4m + 4n = 3$ — ③

& Eq^n ② becomes,

$$\frac{1}{2}m - \frac{1}{2}n = -\frac{1}{8}$$

Multiply each term by 8,

$\therefore$ $8 \times \frac{1}{2}m - 8 \times \frac{1}{2}n = 8 \times \frac{-1}{8}$

$\therefore$ $4m - 4n = -1$ — ④

Add eq^n ③ & eq^n ④,

$$4m + \cancel{4n} = 3$$
$$+$$
$$4m - \cancel{4n} = -1$$
$$\rule{6cm}{0.4pt}$$
$$8m \quad = \quad 2$$

$\therefore$ $m = \dfrac{2}{8}$

$$\therefore \quad \boxed{m = \frac{1}{4}}$$

Put $m = \frac{1}{4}$ in eq? ⑪,

$$4m + 4n = 3$$

$$\therefore \quad \left(4 \times \frac{1}{4}\right) + 4n = 3$$

$$\therefore \quad 1 + 4n = 3$$

$$\therefore \quad 4n = 3 - 1$$

$$\therefore \quad 4n = 2$$

$$\therefore \quad n = \frac{2}{4}$$

$$\therefore \quad \boxed{n = \frac{1}{2}}$$

But, $m = \dfrac{1}{3x+y}$ & $n = \dfrac{1}{3x-y}$

$$\frac{1}{4} = \frac{1}{3x+y} \quad \& \quad \frac{1}{2} = \frac{1}{3x-y}$$

$$\therefore \quad 3x + y = 4 \quad — \text{(V)}$$

$$\& \quad 3x - y = 2 \quad — \text{(VI)}$$

Add eqⁿ (V) & eqⁿ (VI),

$$
\begin{aligned}
3x + y &= 4 \\
+ \quad 3x - y &= 2 \\
\hline
6x &= 6
\end{aligned}
$$

$$\therefore \quad x = \frac{6}{6}$$

$$\therefore \quad \boxed{x = 1}$$

Put $x = 1$ in eqⁿ (V).

$$3x + y = 4$$

$$\therefore \quad (3 \times 1) + y = 4$$

$$\therefore \quad 3 + y = 4$$

$$\therefore \qquad y = 4 - 3$$

$$\therefore \qquad \boxed{y = 1}$$

$\therefore$ The solution is,

$$(x, y) = (1, 1).$$